An Algebraic Method for Compensating for Coil-Placement Errors in Three-Dimensional Search-Coil Eye Trackers

W. James MacLean & Richard C. Frecker

Institute of Biomedical Engineering and Department of Electrical Engineering, University of Toronto
Toronto, Canada M5S 1A1

ABSTRACT

A new method for compensating for coil-placement errors in three-dimensional scleral search-coil eye trackers has been developed. The position of the coils may be measured exactly and simply. The error-compensation method avoids difficult transcedental equations by expressing eye position as a linear combination of coil position vectors. The coil position vectors are considered to form a rigid, non-orthogonal basis in the frame of reference of the globe. The constants relating eye position to this basis may be derived from a calibration session.

INTRODUCTION

A standard method of measuring eye movements using a scleral induction search coil system was developed by Robinson [1963]. This system allows three dimensional measurement of eye movements (yaw, pitch, and roll). Our frame of reference is a right-handed system where the primary position of the eye lies in the **x** direction and **z** is up. Therefore pure yaw takes place in the X-Y plane and pure pitch in the X-Z plane.

The subject sits in two magnetic fields in quadrature, one in the **y** direction and one in the **z** direction, and eye position is measured from the induced voltages in a coil placed on the eye, usually embedded in a contact lens. To measure torsion a second coil is used. The system is non-linear: eye position may be related to coil voltages by the relations

$$
\begin{aligned}
E_\Theta &= K_\Theta \cdot \sin\Theta \cdot \cos\phi \\
E_\phi &= K_\phi \cdot \sin\phi \\
E_\Phi &= K_\Phi \cdot \cos\phi \cdot \sin\Phi
\end{aligned}
\qquad \text{Eq 1}
$$

where Θ = yaw, ϕ = pitch, and Φ = roll and the K's are proportionality constants. When the constants of proportionality are known it is a simple matter to deduce Θ, ϕ, and Φ from E_Θ, E_ϕ, and E_Φ. These equations assume that the coils are perfectly placed on the eye, which is to say that the normal vector of the front coil coincides with the visual axis, and that the torsion coil's normal vector is colinear with the **y** axis when the eye is in the primary position. In the event that the coils are imperfectly placed a method exists to correct for placement errors [Ferman et al, 1987]. It expresses the coil voltages in terms of transcendental equations in Θ, ϕ, and Φ and Θ_0, ϕ_0, and Φ_0 (the coil placement errors).

This paper suggests an algebraic approach to correcting for coil-placement errors that greatly simplifies the procedure by working in cartesian coordinates until the final step, at which point the relations in Eq 1 suffice.

CORRECTING FOR COIL PLACEMENT ERRORS

The coils are perceived to have placement errors due to the fact that the normal vector of the search coil does not coincide with the visual axis of the eye. The discrepancy can be modelled as a rotation of the coil through angles Θ_0, ϕ_0, and Φ_0. However, with proper calibration it is possible to measure coil position accurately (as opposed to eye position) using the relations of Eq 1. Once coil positions are known they are expressed as 3-vectors in cartesian coordinates. It then remains to generate a relationship between coil position and eye position: this will be a fixed relationship since nominally the coils do not move with respect to the globe[1]. Therefore measuring eye position involves measuring coil position with respect to a fixed frame of reference (as defined by the magnetic fields), and then expressing eye position as a function of coil position. The coil position vectors can be used to form a rigid basis set in the frame of reference of the globe. In general this basis set is non-orthogonal.

We denote the coil vectors by $\mathbf{a}_F$ (a vector normal to the front coil's surface and proportional to its area) and $\mathbf{a}_T$ (normal to the torsion coil's surface and normal to its area). In practice there are two types of torsional coils used - the first employs a secondary coil in the front coil which is effectively wound in the sagittal plane and the second employs a second coil mounted on the side of the globe. Both types make use of a second coil which nominally is orthogonal to the front coil. In the case of a second, side-mounted coil two sets of coil placement errors must be accounted for.

We will define three vectors: the visual axis **f**, torsion vector **t**, and torsion reference **t'**. The direction of **f** represents the line of sight, and **t** is defined in such a way that **t** f and **t** lies in the X-Y plane when the eye is in the primary position. **t'** is defined such that **t'** **f** and **t'** <u>always</u> lies in the X-Y plane. Torsion angle Φ is then defined by the angle between **t** and **t'** at any point in time.

CH2770-6/89/0000-0733 $01.00 C 1989 IEEE

In ideal search-coil placement $\mathbf{a}_F$ and $\mathbf{f}$ are colinear and $\mathbf{t}$ and $\mathbf{a}_T$ are colinear. This is seldom the case, but the orientation of $\mathbf{f}$ and $\mathbf{t}$ to $\mathbf{a}_F$ and $\mathbf{a}_T$ is fixed as both sets of vectors are fixed in the frame of reference of the globe.

Upon determining the coil positions $\mathbf{a}_F$ and $\mathbf{a}_T$ actual eye position may be determined as a linear combination of the basis formed by $\mathbf{a}_F$, $\mathbf{a}_T$, and ($\mathbf{a}_F \times \mathbf{a}_T$) as follows

$$\mathbf{f} = \alpha_1 \mathbf{a}_F + \alpha_2 \mathbf{a}_T + \alpha_3(\mathbf{a}_F \times \mathbf{a}_t)$$

$$\mathbf{t} = \beta_1 \mathbf{a}_F + \beta_2 \mathbf{a}_T + \beta_3(\mathbf{a}_F \times \mathbf{a}_t) \quad \text{Eq 2}$$

The coefficients α_i and β_i may be determined by recording values of $\mathbf{a}_F$ and $\mathbf{a}_T$ for known eye positions. The coefficients are a function of coil placement error. For perfect coil placement we have

$$\alpha_1 = k_1 \quad \alpha_2 = 0 \quad \alpha_3 = 0$$

$$\beta_1 = 0 \quad \beta_2 = k_2 \quad \beta_3 = 0 \quad \text{Eq 3}$$

The constants α_3 and β_3 are necessary as $\mathbf{f}$ and/or $\mathbf{t}$ may lie outside of the plane defined by $\mathbf{a}_F$ and $\mathbf{a}_T$.

Eye position, corrected for coil placement errors, is given by

$$\Theta = \sin^{-1}[f_x/(\|\mathbf{f}\| \cos\phi)]$$

$$\phi = \sin^{-1}[f_z/\|\mathbf{f}\|]$$

$$\Phi = \cos^{-1}[\mathbf{t} \cdot \mathbf{t}'/(\|\mathbf{t}\| \, \|\mathbf{t}'\|)] \quad \text{Eq 4}$$

The sign of Φ can be defined knowing the components of $\mathbf{t}$ and $\mathbf{t}'$. It is easy to reconstruct $\mathbf{t}'$ as we know that $\mathbf{t}' \cdot \mathbf{f} = 0$ and $t'_z = 0$. The magnitude of $\mathbf{t}'$ is of no concern to us, so we set $t'_y = 1$, therefore $t'_x = -f_y/f_x$ and $\mathbf{t}' = [-(f_y/f_x) \;\; 1 \;\; 0]^T$.

This method involves an exact determination of the position of $\mathbf{a}_T$. Many coil systems only collect pitch information from the torsion coil - in this method it is necessary to have yaw information in order to locate $\mathbf{a}_T$ in space. Direct measurement of yaw for the torsion coil presents a problem in that the torsion coil yaw signal lies on the cosine part of the curve (Eq 1) and as such varies little for different yaw positions of $\mathbf{a}_T$. One possible solution would be to add a third field in frequency quadrature in the $\mathbf{x}$ direction to discern information about $\mathbf{a}_T$. A second possibility would be to make simplifying assumptions about the yaw of the torsion coil based on the yaw of the front coil[2].

The coil position vectors $\mathbf{a}_F$ and $\mathbf{a}_T$ need not be orthogonal for this method to work. The relative values of α_i and β_i will reflect on the relative position and calibration of the two coils. As a result another solution exists to the problem of determing yaw position for $\mathbf{a}_T$, simply mount the side coil farther forward on the globe, perhaps 45° forward. In this manner yaw information about $\mathbf{a}_T$ no longer suffers from the desensitivity caused by residing on the cosine portion of the curve. It is not necessary to place the side coil at 90° to the front coil.

CONCLUSIONS

It is possible to account for coil placement error in three-dimensional scleral search coil systems using a linear correction method in cartesian coordinates. Coil position for front and torsion coils is measured and expressed as a 3-vector in cartesian space. Coil position vectors are then used as a basis set to express eye position. The constants relating coil position to eye position may be determined in a calibration session.

REFERENCES

Ferman L, Collewijn H, Jansen TC, Van den Berg AV. **Human gaze stability in the horizontal, vertical, and torsional direction during voluntary head movements, evaluated with a three-dimensional scleral induction coil technique.** Vision Res 1987 27(5):811-828

Robinson DA. **A method of measuring eye movements using a scleral search coil in a magnetic field.** IEE Trans Biomed Electronics 1963 10:137-145

---

1 In many experimental situations search coils are sutured directly to the animal's eye, thereby ensuring that no slippage occurs. With human subjects it will be necessary to consider the problem of slippage.

2 Since the torsion coil has a fixed area and a fixed position with respect to the front coil, it is possible to discern yaw information for the side coil as a function of front coil position.

W. James MacLean
Institute of Biomedical Engineering
University of Toronto,
Toronto, CANADA
M5S 1A1

416-978-2236